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CHIRAL RADIATIVE CORRECTIONS IN POTENTIAL MODEL AND THE P-WAVE STATES OF HEAVY-LIGHT MESONS

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In relativistic potential model of heavy-light mesons it is essential to incorporate the chiral radiative corrections before comparing the model predictions with data. Once the radiative corrections are taken into account the potential model can explain the unusual mass spectra and absence of the spin orbit-inversion in P-wave states.

Keywords: chiral corrections; heavy-light mesons; spin-orbit inversion.

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1. Introduction

It was noted from the very moment of the first observation of a P-wave state of heavy-light mesons that the relativistic potential model was in trouble in explaining the data^{1,2}. For instance, the mass of $D_s(2317)$ was too low compared to its nonstrange counterpart $D(2308)$ and because of this the potential model could not explain the observed mass gap

$$\text{gap} \equiv [m(D(0^+)) - m(D(0^-))] - [m(D_s(0^+)) - m(D_s(0^-))] \approx 95 \text{ MeV} \quad (1)$$

which in the potential model is virtually vanishing.

Another puzzle is the absence of the spin-orbit inversion in P-wave states that has been a generic prediction of potential models. Schnitzer suggested long ago that the strong spin-orbit interaction of the scalar confining potential would lead to spin-orbit inversion in P-wave heavy-light mesons, with the claim that its observation would confirm the scalar nature of the confining potential³. This spin-orbit inversion was later reaffirmed in studies with more sophisticated potential models^{4,5}. However, contrary to these studies, the observed masses of the P-wave charmed mesons do not exhibit spin-orbit inversion.

Because of the apparent failure of the potential model various alternatives were suggested⁶. It is worth noting, however, that the conventional potential model had not treated the chiral loop corrections consistently. The radiative correction to the mass of a resonance is given by the real part of the self energy, and this was not taken into account in the calculation of the mass spectra, while the imaginary

part was used in computing the decay width. Since the widths are generally a few hundred MeVs it can be seen immediately that the loop corrections to the mass spectra cannot be ignored.

In this note we review the chiral loop corrections in the potential model and show that the puzzles of the P-wave states can be answered within the potential model once the loop corrections are taken into account.

2. Chiral Radiative Corrections

The one loop corrections for S and P-wave states with $j = \frac{1}{2}$ to estimate the mass gap was computed in Ref. ⁷ and the calculation was extended to P-wave states with $j = 3/2$ as well as D-wave states with $j = 3/2$ and $5/2$ in Ref. ⁸ to solve the spin-orbit inversion problem.

The loop corrections in the potential model which is based on the chiral quark model are UV divergent and in Refs. ^{7,8} a three-momentum cutoff regularization was used. While the loop corrections to the energy levels are cutoff dependent the mass gap, which depends on the mass differences of the resonances, is virtually independent on the cutoff since only the low energy fluctuations of $k \approx 250$ MeV contribute to it.

With the UV cutoff at 700 MeV the loop corrections for the energy levels for S and P-wave states are given as in Table 1.

Table 1. Loop corrections $\delta E_{l,j,q}^{\text{loop}}$ for S and P-wave states in the lowest radial excitations. (Units are in MeV.)

$\delta E_{0,\frac{1}{2},d}^{\text{loop}}$	$\delta E_{0,\frac{1}{2},s}^{\text{loop}}$	$\delta E_{1,\frac{1}{2},d}^{\text{loop}}$	$\delta E_{1,\frac{1}{2},s}^{\text{loop}}$	$\delta E_{1,\frac{3}{2},d}^{\text{loop}}$	$\delta E_{1,\frac{3}{2},s}^{\text{loop}}$
-161	-160	-260	-343	-183	-181

3. Mass Gap

We can now see how the mass gap is affected by the radiative corrections. For this we focus on the effects of the loop corrections on the existing potential model predictions in Ref. ⁹.

The new energy levels, denoted by $\bar{E}_{\mathbf{m}}$, that include the radiative corrections can be related to the energy levels of the conventional potential model by

$$\bar{E}_{\mathbf{m}} = E_{\mathbf{m}} + \delta E_{\mathbf{m}}^0 + \delta E_{\mathbf{m}}^{\text{loop}} \quad (2)$$

where $E_{\mathbf{m}}$ denotes the conventional energy levels which contain the leading order level $E_{\mathbf{m}}^0$ as well as the $1/M$ corrections, and $\delta E_{\mathbf{m}}^0$ denotes the shift in the leading order level $E_{\mathbf{m}}^0$ caused by the shift in the fitted values of the parameters of the model, which was induced by the introduction of radiative corrections $\delta E_{\mathbf{m}}^{\text{loop}}$ in the fitting of the parameters. The relation (2) is valid to the leading order of the loop corrections.

Now noting that the gap for the leading order levels is vanishing we expect the correction to it to be small and so obtain approximately

$$\text{gap}^{\text{new}} \approx \text{gap}^{\text{old}} + \text{gap}^{\text{loop}} = 72 \text{ MeV} \quad (3)$$

which is consistent with the experimental value 95 MeV.

4. Spin-Orbit Inversion

Now applying the relation (2) to states with differing j but otherwise same quantum numbers in the heavy-quark limit we obtain approximately

$$\bar{E}_j - \bar{E}_{j'} = E_j - E_{j'} + \delta E_j^{\text{loop}} - \delta E_{j'}^{\text{loop}}. \quad (4)$$

Let us focus on the spin-orbit inversions in P-wave states. Looking on Table 1 we notice that the magnitudes of the loop corrections are larger for states with smaller j , and this feature will be crucial for understanding the absence of spin-orbit inversions.

We shall first consider the effects of the loop corrections on the P-wave D_s mesons. In the following all the states, labeled $H(l, j, J)$, are in their lowest radial excitations. We shall assume that the modified energy level for the state $D_s(1, \frac{1}{2}, 0)$ coincides with the experimental mass of $D_s(2317)$, and then estimate the masses of $j = 1/2$ and $3/2$ states. Reading the values of the conventional energy levels from Ref. ⁹ and loop corrections from Table 1 we find the following new energy levels of the states related by Eq. (4):

$$\begin{aligned} \bar{E}_{D_s(1, \frac{1}{2}, 1)} &= 2435 \text{ MeV}, \\ \bar{E}_{D_s(1, \frac{3}{2}, 1)} &= 2527 \text{ MeV}, \\ \bar{E}_{D_s(1, \frac{3}{2}, 2)} &= 2573 \text{ MeV}. \end{aligned} \quad (5)$$

Comparing this result with the experimental values 2460 MeV, 2535 MeV, and 2573 MeV, respectively, we see there is good agreement between the new levels and data, and there are no longer spin-orbit inversions.

Using the same procedure we can obtain the modified energy levels for P-wave D mesons as well, and the result is summarized in Table 2.

We can now use Eq. (2) to estimate the masses of the P-wave bottom mesons. We shall first compute the mass for $B_s(1, \frac{1}{2}, 0)$, which is the counterpart of $D_s(2317)$. Since the loop corrections are independent of the heavy quark mass we see that the last two terms in Eq. (2) to be heavy-quark mass independent, so we get

$$\bar{E}_{B_s(l, j, J)} - E_{B_s(l, j, J)} = \bar{E}_{D_s(l, j, J)} - E_{D_s(l, j, J)}. \quad (6)$$

Identifying again $\bar{E}_{D_s(1, \frac{1}{2}, 0)$ with the mass of $D_s(2317)$ we find $\bar{E}_{B_s(1, \frac{1}{2}, 0)} = 5634$ MeV. With this energy level we can then compute the levels of other P-wave states. The result is summarized in Table 3. An interesting feature of our estimation is that for $j = 1/2$ the B_s mesons have almost equal or slightly smaller masses than their non-strange counterparts.

Table 2. Modified energy levels \bar{E} for P-wave charmed mesons. (Units are in MeV.)

	$D(\frac{1}{2}, 0)$	$D(\frac{1}{2}, 1)$	$D(\frac{3}{2}, 1)$	$D(\frac{3}{2}, 2)$	$D_s(\frac{1}{2}, 0)$	$D_s(\frac{1}{2}, 1)$	$D_s(\frac{3}{2}, 1)$	$D_s(\frac{3}{2}, 2)$
$m_{\text{ex.}}$	2308	2427	2422	2459	2317	2460	2535	2573
\bar{E}	2308	2421	2425	2468	2317	2435	2527	2573
E	2377	2490	2417	2460	2487	2605	2535	2581

Table 3. Modified energy levels \bar{E} for P-wave bottom mesons. (Units are in MeV.)

	$B(\frac{1}{2}, 0)$	$B(\frac{1}{2}, 1)$	$B(\frac{3}{2}, 1)$	$B(\frac{3}{2}, 2)$	$B_s(\frac{1}{2}, 0)$	$B_s(\frac{1}{2}, 1)$	$B_s(\frac{3}{2}, 1)$	$B_s(\frac{3}{2}, 2)$
\bar{E}	5637	5673	5709	5723	5634	5672	5798	5813
E	5706	5742	5700	5714	5804	5842	5805	5820

5. Conclusion

Chiral loop corrections in potential model cannot be ignored since these are not small, for instance in charmed mesons they are comparable to the $1/M_c$ corrections. However, these were completely ignored in the computations of energy levels in conventional potential model. Once the chiral loop corrections are taken into account we find important puzzles such as the mass gap and spin-orbit inversion can be understood within the potential model. This suggests that the P-wave states be of more conventional type rather than exotic states such as four-quark states.

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